RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2017

FIRST YEAR [BATCH 2016-19]

Date : 22/05/2017 **MATHEMATICS (General)**

Time : 11 am – 2 pm **Paper : II** Full Marks : 75

[Use a separate Answer Book for each group]

Group - A

Answer **any three** questions from **Question Nos. 1 to 5** : [3×5]

1. If by a rotation of rectangular axes about the origin, $(ax+by)$ and $(cx+dy)$ be changed to $(a'x'+b'y')$ and $(c'x'+d'y')$ respectively, show that $ad-bc=a'd'-b'c'$. 5

2. a) Find the equation of the bisectors of the angle between the lines $x^2-4xy-y^2=0$. 3
   b) Find the angle between the pair of straight lines $y^2+xy-2x^2-5x-y-2=0$. 2

3. Discuss the nature of the conic represented by $9x^2-24xy+16y^2-18x-101y+19=0$ and reduce it to the Canonical form. 5

4. Two tangents drawn to the parabola $y^2=4ax$ meet at an angle 45°. Show that the locus of their point of intersection is $(x+a)^2=y^2-4ax$. 5

5. Show that for the conic $\frac{l}{r}=1+e\cos\theta$, the equation to the directrix corresponding to the focus other than the pole is $\frac{l}{r}=-\frac{1-e^2}{1+e^2}e\cos\theta$. 5

Answer **any three** questions from **Question Nos. 6 to 10**: [3×5]

6. In any triangle ABC, with usual notations, prove that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$. 5

7. Find the moment about the point $(\hat{i}+2\hat{j}-\hat{k})$ of a force represented by $(3\hat{i}+\hat{k})$ acting through the point $(2\hat{i}-\hat{j}+3\hat{k})$. 5

8. a) In a triangle ABC on a plane, let $\overrightarrow{BC}=\vec{a}, \overrightarrow{CA}=\vec{b}$ and $\overrightarrow{AB}=\vec{c}$. Prove, by vectors, that $\cos A=\frac{b^2+c^2-a^2}{2bc}$ where *a, b, c* are lengths of the sides $\overline{BC}, \overline{CA}, \overline{AB}$ respectively. 2
   b) Show that perpendiculars from the vertices of a triangle to the opposite sides are concurrent. 3

9. Given two vectors $\vec{\alpha}=3\hat{i}-\hat{j}$ & $\vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$, express $\vec{\beta}$ in the form $\vec{\beta}_1+\vec{\beta}_2$, where $\vec{\beta}_1 \parallel \vec{\alpha}$ and $\vec{\beta}_2 \perp \vec{\alpha}$. 5

10. Show that $\left[\vec{b}\times\vec{c}, \vec{c}\times\vec{a}, \vec{a}\times\vec{b}\right] = \left[\vec{a}\vec{b}\vec{c}\right]^2$. 5

Group - B

Answer **any five** questions from **Question Nos. 11 to 18** : [5×5]

11. If $\lim\limits_{x\to 0}\dfrac{\sin 2x + a\sin x}{x^3}$ be finite, find the value of a and the limit. 2+3

12. Find the asymptotes of the curve $x^2y - x^2 - y^3 = 0$. 5

13. Find the envelope of the family of curves, with parameter α, given by $(x-\alpha)^2 + (y-\alpha)^2 = 2\alpha$. 5

14. Find the greatest and the least values of the function $f:[0,4]\to\mathbb{R}$ defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$. 5

15. Prove that every convergent sequence is bounded, but the converse is not necessarily true. Explain with example. 5

16. Test the following two series for convergence:

(a) $\dfrac{1}{2} + \dfrac{2}{2^2} + \dfrac{3}{2^3} + \cdots + \dfrac{n}{2^n} + \cdots$

(b) $\dfrac{1}{3} + \left(\dfrac{2}{5}\right)^2 + \left(\dfrac{3}{7}\right)^3 + \cdots + \left(\dfrac{n}{2n+1}\right)^n + \cdots$ 2½+2½

17. Determine the following function $f(x)$ satisfying the conditions of Rolle's theorem in the given interval and when the required conditions are fulfilled, exhibit at least one value of x in the interval at which $f'(x) = 0$, $f(x) = \sin x\cos x,\ 0 \le x \le \pi/2$. 5

18. Decompose the number 36 into two factors such that the sum of their square is the least possible. 5

Answer **any two** questions from **Question Nos. 19 to 21** : [2×5]

19. a) Evaluate $\displaystyle\int \frac{dx}{5 - 3\cos x}$. 3

b) Prove that $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. 2

20. a) Find $\displaystyle\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\cdots\cdots\left(1+\frac{n}{n}\right)\right\}^{\frac{1}{n}}$. 4

b) Find the value of $\displaystyle\int_{-1}^{1} |x|\,dx$. 1

21. If $J_n = \int_0^{\pi/4} \tan^n x \, dx$ where n is a positive integer, show that $J_n = \frac{1}{n-1} - J_{n-2}$. Use this result to evaluate $\int_0^{\pi/4} \tan^6 x \, dx$. 3+2

Answer **<u>any two</u>** questions from **<u>Question Nos. 22 to 24</u>** : [2×5]

22. a) Solve: $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2 + y^2} = 0$. 3

b) State the order and the degree of the differential equation $\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2} = K\frac{d^2y}{dx^2}$. 2

23. Solve: $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$. 5

24. Obtain the general and singular solutions of $y = px + \sqrt{1 + p^2}$ where $p = \frac{dy}{dx}$. 3+2

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